Ideal behaviour of a *perfect* boomerang

Hugh Hunt
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This analysis applies to any boomerang, but is particularly applied to the cross-shaped boomerang shown on Fig. 1. The analysis follows closely the assumptions of conventional propeller theory. The boomerang sweeps a circular area and every part of the swept circle is "visited" by a segment of the airfoil at a sufficiently-rapid rate so that the swept area can be considered to be a uniform disc with distributed lift.

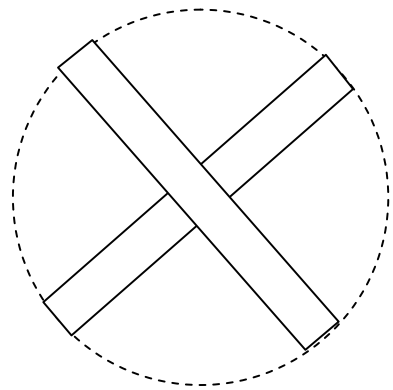

Fig. 1

An element of the lifting disc of area dA is assumed to generate a lift force d$F = \frac{1}{2}\rho v^2 C_L \,\mathrm{d}A$, where C_L is a lift coefficient for the airfoil, ρ is the density of air and v is the local velocity of the element dA of the lifting disc.

It is necessary to determine the elemental velocity v from the kinematics that apply to a boomerang. Fig. 2 shows the forward velocity V and the angular velocity ω of a lifting disc of radius a. The elemental area dA = rdrdθ and the velocity of the element $v = V\sin\theta + a\omega$. It is possible then by integration to compute the total lift force $F = \iint dF$ and the total lift couple $C = \iint r\sin\theta\, dF$.

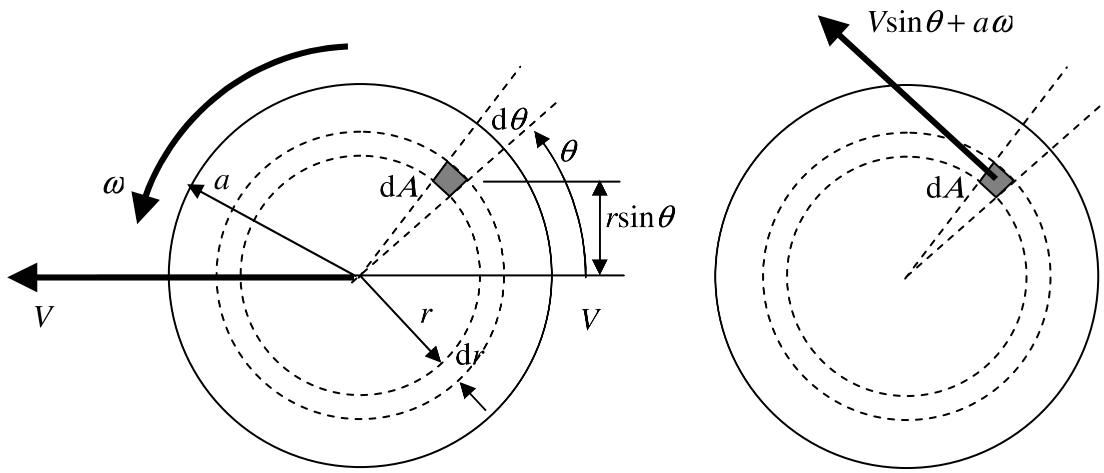

Fig. 2

For the purposes of the following analysis these integrals will be useful:

$$\int_0^{2\pi} d\theta = 2\pi \qquad \int_0^{2\pi} \sin\theta\, d\theta = 0 \qquad \int_0^{2\pi} \sin^2\theta\, d\theta = \pi \qquad \int_0^{2\pi} \sin^3\theta\, d\theta = 0$$

So the total lift force is given by

$$F = \int_0^a \int_0^{2\pi} \tfrac{1}{2}\rho v^2 C_L dA \quad = \int_0^a \int_0^{2\pi} \tfrac{1}{2}\rho (V\sin\theta + r\omega)^2 C_L r\, dr\, d\theta$$

$$= \tfrac{1}{2}\rho C_L \int_0^a \int_0^{2\pi} (V^2 r \sin^2\theta + 2V\sin\theta\, r^2\omega + r^3\omega^2)\, dr\, d\theta$$

$$= \tfrac{1}{2}\rho C_L \int_0^a (\pi V^2 r + 2\pi r^3 \omega^2)\, dr$$

$$= \tfrac{1}{4}\rho C_L (\pi V^2 a^2 + \pi a^4 \omega^2)$$

$$= \tfrac{1}{4}\rho C_L \pi a^2 (V^2 + (a\omega)^2) \qquad (1)$$

and the total aerodynamic couple is given by

$$C = \int_0^a \int_0^{2\pi} \tfrac{1}{2}\rho v^2 C_L r \sin\theta \, dA \quad = \int_0^a \int_0^{2\pi} \tfrac{1}{2}\rho (V \sin\theta + r\omega)^2 C_L r^2 \sin\theta \, dr\, d\theta$$

$$= \tfrac{1}{2}\rho C_L \int_0^a \int_0^{2\pi} (V^2 r^2 \sin^3\theta + 2V \sin^2\theta \, r^3 \omega + r^3 \sin\theta \, \omega^2)\, dr\, d\theta$$

$$= \tfrac{1}{2}\rho C_L \int_0^a 2V\pi r^3 \omega \, dr$$

$$= \tfrac{1}{4}\rho C_L \pi a^4 \omega V \qquad (2)$$

These are the external aerodynamic actions. To determine the response of the boomerang two well-known dynamical equations are used:

$$F = mR\Omega^2 \qquad (3)$$

and

$$C = J\Omega\omega \qquad (4)$$

for steady-state circular motion and gyroscopic precession respectively, where J is the polar moment of inertia of the boomerang. The radius of the flight of the boomerang R is related to the forward velocity V and the turning rate Ω by $V = R\Omega$.

Equations (2) and (4) can now be used to determine the flight radius of the boomerang:

$$R = \frac{4J}{\rho C_L \pi a^4} \qquad (5)$$

and equations (1), (3) and (5) give a further relationship:

$$\frac{ma^2}{J} = 1 + \left(\frac{a\omega}{V}\right)^2 \qquad (6)$$

For the particular case of a cross-shaped boomerang the moment of inertia $J = ma^2/3$ which gives rise to the particular equations for the "flick of the wrist":

$$a\omega = \sqrt{2}V \qquad (7)$$